Secondary 3 & 5 Mathematics - Open Classroom by Awarded Teachers of the Chief Executive's Award for Teaching Excellence

Course ID: PDT020260230

Speakers:

Mr LEE Yan-chuen, Andrew and Mr NG Chun-ngai, Lincoln

(Wah Yan College, Kowloon), awarded teachers of

the Chief Executive’s Award for Teaching Excellence

(Mathematics Education Key Learning Area)

Our vision:

Unleashing Students' Potentials through Real-world Mathematics

We believe that mathematics education should **go beyond rote learning** to cultivate the essential 21st-century skills of creativity, digital literacy, problem-solving, and communication.

Through ***mathematical modelling***, we inspire our students to **view society and the world through a mathematical lens**, igniting their curiosity and empowering them to **contribute to the greater good of the community**.

Major objectives in Math Modelling Lessons:

- **Real-world scenarios** where students apply mathematical concepts to analyze, solve problems, and **make predictions**
- Students develop **critical thinking**, **problem-solving** skills, and an understanding of the **practical applications** of mathematics in various fields
- Enriches the learning experiences by emphasizing **reflection**, **discernment**, **care** for others, social **justice** and **service** to the community. How mathematics can be used to promote positive change and cultivate a sense of **responsibility**, **empathy**, and a **commitment** to using their knowledge for the **betterment** of society

Major focuses on Math Modelling Activities:

- ***Experience:*** Students **explore** math modelling problems that are highly **relevant** to their **daily lives**.
- ***Reflection:*** Students **reflect** on their **personal experiences** and **justify** how their math models fit into real-life scenarios.
- ***Action:*** Students are encouraged to **continue exploring** the math modelling problems after class, using math as a lens to **examine problems in their community**.

Students' Preparedness and Learning Preference

Students' Preparedness	• Most students will do their own practices on textbook examples. • Some of them have tried a mathematical modelling task before and acquired preliminary concept of mathematical modelling. • However, there is a lack of experience for them to understand our society via mathematics.
Learning Preferences	• Most of the students are able and eager to learn non-stereotypical and non-routine problems in mathematics. • Most of them expect to learn something that can enrich their personal development. • Some of them are willing to participate in mathematical modelling activities.

Lesson (1): Financial Planning

Lesson objectives:

At the end of the lesson, students will be able to

Knowledge	● Apply percentages and sequences to real-life contexts ● Analyze how these mathematical concepts underpin broader social and economic issues
Skill	● Develop mathematical skills and problem-solving skills ● Evaluate and justify various options of financial commitments
Attitude	● Value prudent wealth management by planning personal savings ● Nurture information literacies by reading documents such as interest tables

Resources:

- Worksheets (8 pages)
- Mobile device / computer to use spreadsheet software program if necessary

Lesson (1): Financial Planning

`

Task 1.1: Defining the question

In this study, you are going to set up a fund for future use by saving a suitable amount of money (deposits) periodically and determining the optimal values of parameters, like interest rate and years of saving, in order to maximize your fund savings.

Question 1:	What kind of fund are you going to set up? After how many years are you going to use the fund? (Please make the lump-sum at least $100 000 to make the activity meaningful) How much you would like to save ultimately for this fund?

Set (Motivation):
Recall what we have done on previous lesson, how can we optimize our financial growth by balancing our timeline, periodic deposits, and interest rates?

Lesson (1): Financial Planning

Task 1.2: Identifying variables (parameters)

In this study, you will need to consider at least the following parameters:

Variable	Description	Example
A	The amount of the fund you would like to save ultimately	A = 1 000 000 if you want to save \$1 000 000
N	Number of years you would like to save your fund	N = 20 if you want to save for 20 years
P	Periodic deposit (to be deposited at the beginning of each period)	P = 1 000 if you plan to save \$1 000 at each period
R%	The annual interest rate of a certain saving plan	R = 6 if the interest rate is 6% p.a.
n	Number of compounds in a year	n = 4 for compounding quarterly
D	Total value of deposit throughout the saving plan	

Question 2: In addition to the above parameters, do you identify any other parameters? If yes, please briefly describe them below.

Task 1.3: Formulating mathematical model

Now you need to use the above parameters to suggest formulas to find the following quantities:

Question 3: Suppose **initial deposit = \$$P$**. State the formulas to express A in terms of the parameters respectively. Try to express the balance of your fund **at the end** the following time.

- the 1st compounding period:
- the 2nd compounding period:
- the 3rd compounding period:
- the kth compounding period:
- the Nth year:

where $D =$ and $\frac{D}{A} =$

$\frac{D}{A}$ represents the fraction of the fund that comes from your pocket. You may use this ratio as a tool to analyze the "*levering effect*" of compounding your deposits.

Set (Motivation):
Challenge students to think beyond the basic formula. Have them brainstorm real-world financial factors (like inflation, changing interest rates, or tax brackets) that could alter their savings plan for Question 2.

Lesson (1): Financial Planning

Task 2.1: Solving mathematical problems

Question 4: Please state the value A according to your objective in Question 1.

Based on your model, find the values of P with different values of N, with different combinations of R and n. (Correct your answer to the nearest integer.)

A = ________________

N	$R = 5$		$R = 4$	
	$n = 1$	$n = 2$	$n = 1$	$n = 2$
5	$P =$ $D =$	$P =$ $D =$	$P =$ $D =$	$P =$ $D =$
10	$P =$ $D =$	$P =$ $D =$	$P =$ $D =$	$P =$ $D =$
15	$P =$ $D =$	$P =$ $D =$	$P =$ $D =$	$P =$ $D =$
20	$P =$ $D =$	$P =$ $D =$	$P =$ $D =$	$P =$ $D =$
25	$P =$ $D =$	$P =$ $D =$	$P =$ $D =$	$P =$ $D =$
30	$P =$ $D =$	$P =$ $D =$	$P =$ $D =$	$P =$ $D =$

You may try to change R and n slightly by yourself and see how D changes when N changes.

You may also compare the value of $\frac{D}{A}$ when N changes.

- According to ***Question 1***, which values of P, R and N will suit your fund saving objective?
- **Mathematically**, is there an **optimal** "N" in each case?
- Are there any other ways to analyze the optimum N **and** P in a **realistic situation**?

Development:
Apply the compound interest formulas and understand how compound interest accelerates wealth growth over time.

Note:
Students can use AI or excel spreadsheet to help them calculate the values.

Lesson (1): Financial Planning

Task 2.2: Validating solutions

Question 5: Try to share your answer with your classmates with different values of A. Do you all get a similar result?

Task 2.3: Drawing appropriate conclusions

Question 6: Please suggest ways and assumptions to improve your model in order to save money for your future financial need. Base on the $\frac{D}{A}$ (*levering effect*) ratio and other practical considerations, how would you make choices like considering a larger N with smaller P, or vice versa? Any other reflections?

Development:
Facilitate a group discussion on validating their solutions. Have students compare how different target amounts affect their plans. Challenge them to draw conclusions on the most efficient saving strategies by analyzing the ratio.

Note:
Students can use AI or excel spreadsheet to help them calculate the values.

Lesson (1): Financial Planning

Task 3.1: Refining assumptions in the previous model

In the previous part, we set up a mathematical model to find the suitable number of years to save a fund by a given interest rate and compounding period:

$$A=\frac{P(1+i)\left[(1+i)^{nN}-1\right]}{i} \text{ (or } P=\frac{Ai}{(1+i)\left[(1+i)^{nN}-1\right]} \text{); } D=PNn; \quad \frac{D}{A}=\frac{inN}{(1+i)\left[(1+i)^{nN}-1\right]}$$

where

Variable	Description
A	The amount of the fund you would like to save ultimately
N	Number of years you would like to save your fund
P	Periodic deposit (made at the beginning of each compounding period)
$R\%$	The annual interest rate of a certain saving plan
n	Number of compounds in a year
i	The interest rate per period ($i=\frac{R\%}{n}$)
D	Total value of deposit throughout the saving plan
D/A	Total deposit to fund ratio

Question 1:	What assumptions have been made in the previous mathematical model? Please briefly describe them below.

Question 2:	What extra factor(s) or parameter(s) will you consider in order to make the model more realistic?

Set (Motivation):
In Task 1.2, we assumed a perfect scenario, but real-world finance is not that basic. By extending the current model, students need to consider further hidden factors – such as inflation, taxes, or fluctuating interest rates – might impact their final savings goal.

Lesson (1): Financial Planning

Question 3: Suggest parameter(s) to be **refined** and / or **further included** in calculating your fund saving model. State the assumption(s) in determining such parameter, like whether the parameter is to be kept constant or not.

*You are suggested to test and verify the refined / extra parameters **one by one** if you consider more than one parameter.*

Task 3.3: Refining mathematical model

Now you need to include the new / refined parameter(s) with your assumption(s) to suggest formulas and find the following quantities:

Question 4: Suppose **initial deposit = $P**. State the formulas to express A in terms of the parameters respectively. Express the balance of your fund **at the end** of the following time:

- the 1st compounding period:
- the 2nd compounding period:
- the 3rd compounding period:
- the kth compounding period:
- the Nth year:

where $D =$

In the last part, we consider the ratio $\frac{D}{A}$ to analyze the "*levering effect*" of compounding over time. You may consider if it is still reasonable to use the same ratio after modifying your model.

Development:
Building on the idealized model from Task 1.3, students now refine their mathematical models to reflect real-world complexity. In this stage, they identify new parameters, state their assumptions, and derive updated algebraic formulas to calculate the fund balance over successive compounding periods.

Lesson (1): Financial Planning

Task 3.4: Solving mathematical problems

Question 5: Please state the value A according to your objective in Part 1.

Based on your model, find the values of P with different values of N, with different combinations of R and n, with your newly added parameter. (Correct your answer to the nearest integer.)

A = _______________

Newly added parameter: _______________

Again, you may also analyze the value of $\frac{D}{A}$ (or *any other reasonable quantity*) for the "*levering effect*" when N changes.

N	$R = 5$		$R = 4$	
	$P\ (n = 1)$	$P\ (n = 2)$	$P\ (n = 1)$	$P\ (n = 2)$
5	$P =$ $D =$	$P =$ $D =$	$P =$ $D =$	$P =$ $D =$
10	$P =$ $D =$	$P =$ $D =$	$P =$ $D =$	$P =$ $D =$
15	$P =$ $D =$	$P =$ $D =$	$P =$ $D =$	$P =$ $D =$
20	$P =$ $D =$	$P =$ $D =$	$P =$ $D =$	$P =$ $D =$
25	$P =$ $D =$	$P =$ $D =$	$P =$ $D =$	$P =$ $D =$
30	$P =$ $D =$	$P =$ $D =$	$P =$ $D =$	$P =$ $D =$

Development:
Students calculate how changing variable impact the monthly installment and total debt. They then analyze the leverage ratio to critically assess the long-term risk and sustainability of different financial commitments.

Note:
Students can use AI or excel spreadsheet to help them calculate the values.

Lesson (1): Financial Planning

Task 3.5: Validating solution

Question 6: Try to share your answer with your classmates with different values of A and parameters added. Do you all get a similar result?

Task 3.6: Drawing appropriate conclusions

Question 7: Do you think the new model developed is more realistic? Please suggest further ways, parameters to be considered and assumptions in order to further refine your model to be an even more realistic one.

Conclusion and Reflection:
Students cross-examine their financial planning data with peers to validate their solutions. They critically evaluate the realistic limitations of their mathematical models, identifying missing real-world parameters or assumptions (e.g., inflation, fluctuating interest rates, or hidden fees) required to refine their models for accurate personal financial planning.

Lesson (1): Financial Planning

	Reflection in the lesson	Reflection after the lesson
Content-focused reflection	During the in-class discussion, students are encouraged to reflect on • their structural knowledge of geometric sequences and summations which can be used to model complex financial systems. • the assumptions and limitations of static mathematical modeling when neglecting dynamic real-world elements.	Through the further studies, students are encouraged to reflect on • how to extend their spreadsheet models by adding realistic variables like inflation or hidden fees • how to review and improve their calculation formulas based on comparisons with other groups.
Value-focused reflection	During the in-class discussion, students are encouraged to reflect on • their teamwork habits, including diligence and cooperation.	After the class, students are suggested to work and reflect on • how to use mathematics to make long-term financial commitments in the future. • their own spending habits, recognizing the importance of budgeting prudently and starting to save early.

Lesson (2): Ramps in Our Campus

Lesson objectives:

At the end of the lesson, students will be able to

Knowledge	● Apply gradient ratios to calculate exact dimensions of a physical structure ● Construct diagrams to represent geometric ideas
Skill	● Apply mathematical modelling in real-life scenarios ● Translate physical measurements into a viable geometric layout
Attitude	● Demonstrate social awareness by recognizing the importance of inclusive design and accessible infrastructure for individuals with mobility challenges ● Exhibit critical thinking and flexibility when tackling real-world limitations

Resources:

- Worksheets (Part A & B 4 pages per each)
- Mobile device / computer for making use of AI if necessary
- Measuring tools

Lesson (2): Ramps in Our Campus

Introduction

Our campus has many ramps built to ensure that people with disabilities can access facilities independently. Here are some examples:

The ramp in the main entrance of canteen

The ramp besides St. Ignatius Chapel

However, there are still some facilities on our campus that do not have ramps. For example, there are only steps to our basketball courts:

Steps from the drive to basketball courts

Steps from the pitch to basketball courts

Set (Motivation):
How can we apply mathematical concepts like slope and gradient to design optimal ramps that improve campus accessibility for everyone?

Lesson (2): Ramps in Our Campus

Task 1: Defining the question

In this study, you are invited to design a ramp in the construction area below.

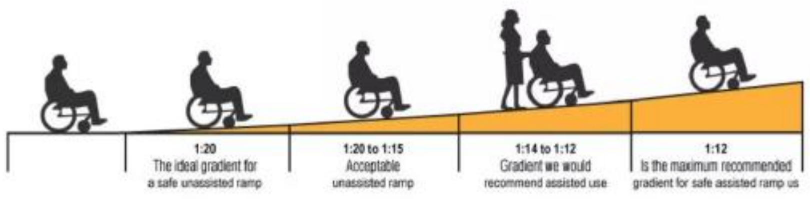
And you are required to find the information below

- Dimension of the construction area: ______ m × ______ m
- Height of the steps from the drive to basketball courts: ______ m
- Height of the steps from the football pitch to basketball courts: ______ m

Development:
Discuss practical methods for measuring the construction area and step heights (Task 1.1). Ask students to work in groups to apply field measurement skills and translate their data into a preliminary ramp model, considering real-world accessibility constraints.

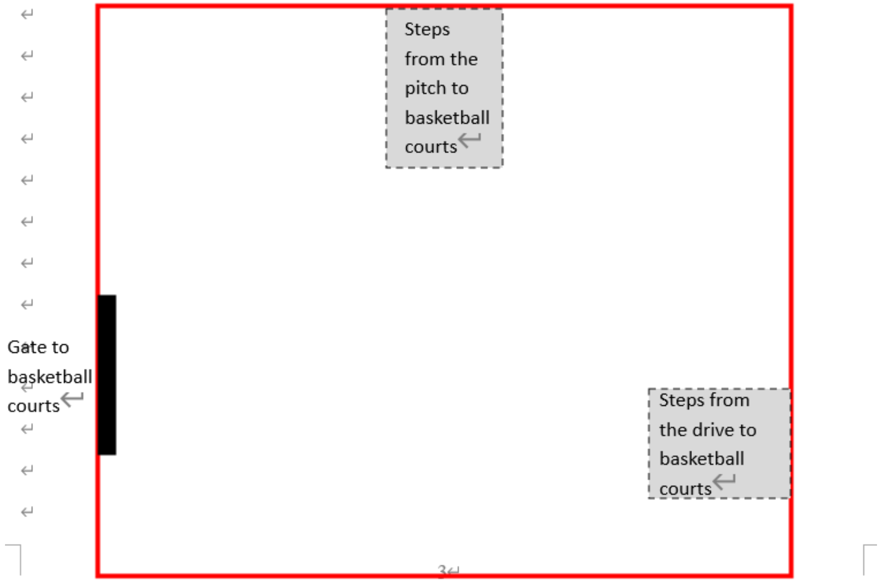
Lesson (2): Ramps in Our Campus

Note that the ramp built should be of gradient less than 1:12 and width of at least 1.05 m.



(Source: https://www.esl-services.co.uk/disability-aids-products/ramp-buying-guide)

Now please design your ramp and draw it on the space on next page, and state clearly the **horizontal distance** and **gradient** of the ramp (if your ramp consists of several parts, please state the horizontal distance and gradient of each part).



Set (Motivation):
Design an inclusive campus ramp by applying mathematical modeling to balance two key real-world constraints:

- **Safety:** Meeting the required 1:12 maximum gradient.
- **Space:** Fitting the design within the limited campus area.

Lesson (2): Ramps in Our Campus

Task 3: Evaluation of the solution

After designing the ramp, you need to estimate **the time required for users with wheelchairs to travel upwards along the ramp.**

According to GenAI, the speed for the users with wheelchairs to travel the ramp upwards with different gradients are given as follows:

Gradient of the ramp	1 : 20	1 : 15	1 : 12
Speed (ms^{-1}) for the users with wheelchairs to travel the ramp upwards	2	1	0.5

Based on your design, estimate the time required (in second) for users with wheelchairs to travel the ramp upwards. Show your work clearly.

Compare the travelling time with your classmates. Is your ramp a good design? Explain your answer.

(Hint: what are the objective parameters justifying for a good design?)

Conclusion and Reflection:

Ask students to evaluate their ramp designs by calculating the estimated travel time using the provided speed-gradient data.

Discuss how their design choices impact real-world usability, balancing a gentler, safer slope against a longer travel time.

Lesson (2): Ramps in Our Campus

	Reflection in the lesson	Reflection after the lesson
Content-focused reflection	During the in-class discussion, students are encouraged to reflect on • applying gradients and trigonometric ratios to calculate slope steepness • evaluating how well their physical layout satisfies standard disability access regulations.	Students are encouraged to reflect on • the possibilities on alternative structural designs and identify realistic layout limitations or safety parameters they might have overlooked during their initial planning.
Value-focused reflection	During the in-class discussion, students are encouraged to reflect on • the group dynamics during the site measurements and layout drafting	After the class, students are suggested to reflect on • the social purpose of mathematics, recognizing their civic responsibility to actively design inclusive, barrier-free campus environments that facilitate independent access for people with physical needs.

Evaluation:

Students will be assessed via

Knowledge	Their works in the worksheet and in-class discussion on how they make use with mathematics to model real-life scenarios
Skill	Their works in the worksheet on how they formulate mathematical model and interpret the result
Attitude	Their engagement in the lesson and their opinion on the usefulness of mathematics in daily life