Secondary 4 & 5 Mathematics - Open Classroom by Awarded Teachers of the Chief Executive's Award for Teaching Excellence

Course ID: PDT020260133

Speakers:

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the Chief Executive’s Award for Teaching Excellence

(Mathematics Education Key Learning Area)

Our vision:

Unleashing Students' Potentials through Real-world Mathematics

We believe that mathematics education should **go beyond rote learning** to cultivate the essential 21st-century skills of creativity, digital literacy, problem-solving, and communication.

Through ***mathematical modelling***, we inspire our students to **view society and the world through a mathematical lens**, igniting their curiosity and empowering them to **contribute to the greater good of the community**.

Major objectives in Math Modelling Lessons:

- **Real-world scenarios** where students apply mathematical concepts to analyze, solve problems, and **make predictions**
- Students develop **critical thinking**, **problem-solving** skills, and an understanding of the **practical applications** of mathematics in various fields
- Enriches the learning experiences by emphasizing **reflection**, **discernment**, **care** for others, social **justice** and **service** to the community. How mathematics can be used to promote positive change and cultivate a sense of **responsibility**, **empathy**, and a **commitment** to using their knowledge for the **betterment** of society

Major focuses on Math Modelling Activities:

- ***Experience:*** Students **explore** math modelling problems that are highly **relevant** to their **daily lives**.
- ***Reflection:*** Students **reflect** on their **personal experiences** and **justify** how their math models fit into real-life scenarios.
- ***Action:*** Students are encouraged to **continue exploring** the math modelling problems after class, using math as a lens to **examine problems in their community**.

Students' Preparedness and Learning Preference

Students' Preparedness	• Most students will do their own practices on textbook examples. • Also, they have tried a mathematical modelling task before and acquired preliminary concept of mathematical modelling. • However, there is a lack of experience for them to understand our society via mathematics.
Learning Preference	• Most of the students are able and eager to learn non-stereotypical and non-routine problems in mathematics. • Most of them expect to learn something that can enrich their personal development. • Some of them are willing to participate in mathematical modelling activities.

Lesson (1): Income and Educational level

Lesson objectives:

At the end of the lesson, students will be able to

Knowledge	Understanding the concept of variation with real-life examples and apply variation to explore social and economic issues
Skill	Apply mathematical modelling in real-life scenarios
Attitude	Appreciate mathematics as a powerful tool for planning and decision making

Resources:

- Worksheets (8 pages)
- Mobile devices / computer to access data in 2021 Population Census if necessary
- Mobile device / computer to use spreadsheet software program if necessary

Lesson (1): Income and Educational level

Introduction

People have always believed that higher levels of education are associated with higher incomes. This is one of the main reasons why we strive to achieve good academic results and go on to university to obtain a bachelor's degree or higher. But have you thought about how we can express the relation between education level and income in terms of mathematics?

The following table shows the median monthly employment earnings of employed persons by educational attainment, provided by the Census and Statistics Department:

Year	Educational level	Level in Qualifications Framework (Q)	Median monthly income (I)
2024 (Q3)	Primary and below	0	13 400
	Lower secondary (F.1-3)	1	16 000
	Upper secondary (F.4-6)	3	19 100
	Post-secondary - diploma/certificate/sub-degree	4	22 000
	*Post-secondary - degree	5 to 7	37 100

Source: https://www.censtatd.gov.hk/en/web_table.html?id=210-06315A

Set (Motivation):
How we can study the relationship between educational level and income through variation?

Lesson (1): Income and Educational level

Task 1.2: Making simple assumptions

To make your study simpler, you can have the following assumption:

- The median monthly income of people in Hong Kong I is only affected by his or her educational attainment Q, where Q is the educational level according to the Qualifications Framework (from 0 to 7).

Source: https://www.hkqf.gov.hk/en/about-hkqf

Set (Motivation):
How can we quantify education level? Can it be done by referring to the Qualification Framework? Why or why not?

Lesson (1): Income and Educational level

Task 1.3: Formulating mathematical model

A students suggested that the relationship between the level of educational attainment in Qualifications Framework (Q) and the median monthly income (I) could be expressed by either one of the following models:

Model	Expression (k is constant)	Description
1	$I = kQ$	Monthly income varies directly as the educational level
2	$I = k_1 + k_2Q$	Monthly income is partly constant and partly varies directly as the educational level
3	$I = k_1 + k_2Q^2$	Monthly income is partly constant and partly varies directly as the square of the educational level

Question 2: After studying the data about Q and I, a student points out that Model 1 ($I = kQ$) is not suitable for modelling the data. Do you agree? Explain your answer.

Q	0	1	3	4	6
I	13 400	16 000	19 100	22 000	37 100

Development:
Guide students to justify possible relationship (models) between educational level and income.

Lesson (1): Income and Educational level

Question 3:

Which of the above models (2 or 3) best describes the relationship between the level of education in the Qualifications Framework (Q) and the median monthly income (I)? Show your working clearly.

(Hint: In both models, you may set $k_1 = 13\ 400$ and try to solve k_2 by using any one datum in the table. After finding the values of k_2, try to substitute the rest of the data into the models to see whether they fit the models or not.)

Question 4: Using your model, estimate the median monthly income of people with a bachelor's, master's and doctoral degree respectively.

Question 5: Can your model provide a good estimation of the median monthly income of people with a bachelor's, master's and doctorate degree? Why (or why not)?

(Hint: You can find the average salaries of people with a bachelor's, master's and doctoral degrees on the internet and compare them with your estimation.)

Development:
Ask students to divide into groups and study on the possible models, then justify who model best describe the relationship between educational level and income in Hong Kong.

Note:
Students can use linear regression software or GenAI to help them set up models.

Lesson (1): Income and Educational level

District	Total population	Sex ratio (the number of males per 1 000 females)	Median age	% Population of Chinese	% Population aged 15 and over and attained Bachelor Degree or above	Average domestic household size	Median monthly domestic household income
	P	*R*	*A*	*C*	*E*	*S*	*I*
Central and Western	235 953	714	44.8	78	42	2.6	45,040
Wan Chai	166 695	673	46.0	76	42	2.6	45,000
Sai Kung	489 037	839	44.7	90	30	2.9	37,840
Southern	263 278	791	48.1	86	26	2.9	32,440
Tsuen Wan	320 094	843	45.4	93	27	2.8	32,040
Eastern	529 603	789	49.0	91	29	2.8	32,000
Kowloon City	410 634	777	45.4	89	30	2.7	30,010
Tai Po	316 470	857	45.7	93	24	2.8	30,000
Islands	185 282	847	42.7	78	27	2.7	28,960
Sha Tin	692 806	848	46.2	94	25	2.8	28,870
Yuen Long	668 080	887	43.7	93	20	2.8	27,560
Yau Tsim Mong	310 647	808	44.0	83	29	2.5	26,080
Tuen Mun	506 879	902	46.1	95	17	2.6	25,040
Kwai Tsing	495 798	881	48.0	95	16	2.7	23,740
North	309 631	885	46.3	96	18	2.7	23,580
Wong Tai Sin	406 802	855	50.1	96	17	2.7	23,520
Kwun Tong	673 166	865	48.0	96	17	2.7	22,000
Sham Shui Po	431 090	848	46.2	94	21	2.5	21,490

Source: https://www.census2021.gov.hk/en/district_profiles.html

Set (Motivation):
Can we extend our models to explain how domestic household income is affected by other factors in our community?

Lesson (1): Income and Educational level

District	Total population	Sex ratio (the number of males per 1 000 females)	Median age	% Population of Chinese	% Population aged 15 and over and attained Bachelor Degree or above	Average domestic household size	Median monthly domestic household income
	P	*R*	*A*	*C*	*E*	*S*	*I*
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Sai Kung	489 037	839	44.7	90	30	2.9	37,840
Southern	263 278	791	48.1	86	26	2.9	32,440

Question 2:

Based on your preliminary observations, propose a mathematical model that explains how the median monthly household income is affected by other factors.

Guidelines:

Step 1: Choose the factor(s) that you think will affect the median monthly domestic household income in the districts.

Step 2: Suggest how the median monthly domestic household income (I) varies with the factor(s) you have chosen (e.g. linear or exponential?)

Step 3: Test your mathematical model using the data in the table. It is recommended to use a spreadsheet program (e.g. MS Excel) or online regression applet to reduce your workload in computation.

Step 4: State the assumptions you have made in the model, i.e. in what condition(s) your model is valid.

Development:

1. Ask students to divide into groups and suggest the possible factors that can affect the domestic household income.
2. Ask students to propose a possible model to explain how domestic household income is affected by other factors with formulas (models). e.g.
   $I = k_1 + k_2E$ or $I = k_1 + k_2RE$

Note:

Students can use linear regression software or GenAI to help them set up models.

Lesson (1): Income and Educational level

Now some scholars suggest the following plans **to increase the median monthly domestic household income** of Sham Shui Po district:

Plan A:

Promote Youth Hostel Scheme in Sham Shui Po so that working youths who are Hong Kong permanent residents aged 18 to 30 are eligible to apply as tenants. This plan will decrease the median age of population in Sham Shui Po so that the median monthly domestic household income will be increased.

Plan B:

Implement redevelopment plan and relocate some residents who live in old and dilapidated buildings (e.g. buildings aged 50 or above) in Sham Shui Po to other developing districts such as Northern Metropolis. This plan will decrease the total population of Sham Shui Po so that the median monthly domestic household income will be increased.

Plan C:

Promote Strive and Rise Programme in Sham Shui Po to support students from underprivileged families, support students to continue their studies at foundation diploma, associate degree and degree level. This plan will increase the educational attainment of residents in Sham Shui Po in so that the median monthly domestic household income will be increased.

Question 3: Try to comment on each plan based on your mathematical model.
(i.e. Will the above plans work? Why or why not?)

Development:
Discuss the possible action(s) that we can do to relief the poverty problem in some districts with the lens of mathematical modelling.

Lesson (1): Income and Educational level

	Reflection in the lesson	Reflection after the lesson
Content-focused reflection	During the in-class discussion, students are encouraged to reflect on • how mathematical knowledge can be used to understand our society and its implications, • the assumption and limitation of mathematical modelling, e.g. it uses big data to analysis the real world in macro perspective but neglecting individual differences.	Through the further studies, students are encouraged to reflect on • how to extend their experiences in the lesson to continue exploring their living district using mathematics.
Value-focused reflection	During the in-class discussion, students are encouraged to reflect on • their social awareness, e.g. general economics status in different districts.	After the class, students are suggested to work and reflect on • how we can make use of knowledge to make better decision and take actions that could contribute to the greater good.

Lesson (2): Probabilities about Penalty Kick in Football

Lesson objectives:

At the end of the lesson, students will be able to

Knowledge	• Understanding the concept of conditional probability (with dependent events and mutually exclusive events) • Apply the concept of conditional probabilities in real-life context (studying the choices of zones of goal in shooting a penalty kick on the football pitch)
Skill	Develop and improve mathematical modelling skills such as developing mathematical models, solving mathematical problems and interpreting results
Attitude	Appreciate how to use mathematics to formulate and solve problems in daily life which could facilitate our better decision making

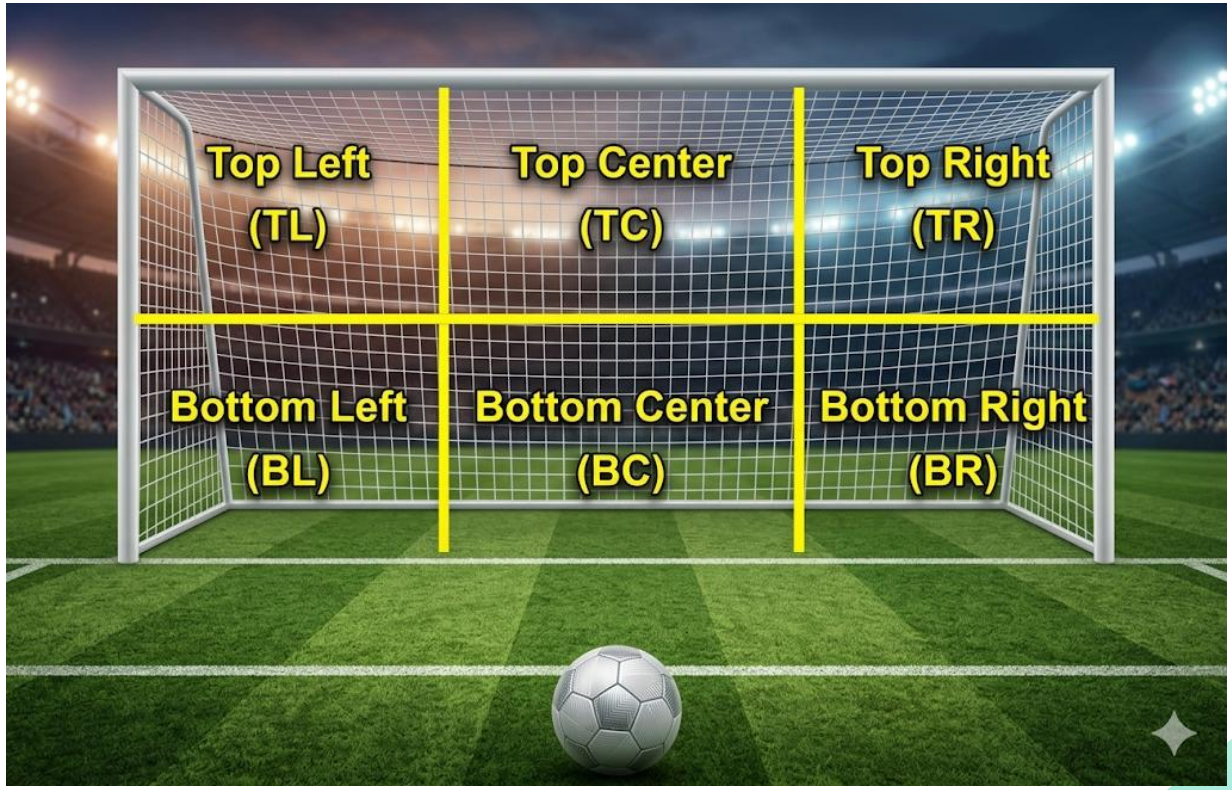
Resources:

- Worksheets (first 4 pages)

Lesson (2): Probabilities about Penalty Kick in Football

Reflection:

When taking a penalty kick, which of the above zones would you prefer to aim at? Why?



Set (Motivation):
How can we quantify the probability of scoring a goal with different zones in a penalty kick?

Lesson (2): Probabilities about Penalty Kick in Football

Question 1(a):

If you aim at the top right zone to shoot the ball, what is the probability that you miss the shot?

Hints:

- You may use the dimension of the top right zone (2.44m × 1.22m) and radius of the ball (11cm) to calculate the ***geometric probability*** of missing the shot.
- You can neglect some factors that are difficult to be quantified (e.g. your shooting skill). However, if you can include these factors in your calculation, you are welcome to do so.

Question 1(b):

Suggest the probability of missing the shot (hit the bar / post or into row Z) when you aim at the following zone in a penalty kick.

Top Left: _____	Top Center: _____	Top Right: _____
Bottom Left: _____	Bottom Center: _____	Bottom Right: _____

Development:

Discuss possible ways of solving Question 1(a) such as

- Geometric probability (area model)
- Experimental probability

Ask students to work in groups to formulate their own solution

(If necessary, students can use BYOD to access GenAI tools to seek assistance)

Lesson (2): Probabilities about Penalty Kick in Football

According to research, ***if the goalkeeper dives in the same direction as your shot***, the probability of your shot being saved is as follows:

Top Left: 20%	Top Center: 50%	Top Right: 25%
Bottom Left: 45%	Bottom Center: 100%	Bottom Right: 45%

Reference: https://sites.northwestern.edu/nusportsanalytics/2020/12/04/the-numbers-behind-penalty-shootout-in-world-cup/

Set (Motivation):
How can we find the probabilities of the possible events of shooting a penalty kick:

- Goals
- Missed
- Saved

And which zone provides the greatest chance for us to score a goal in a penalty kick?

Lesson (2): Probabilities about Penalty Kick in Football

Question 2(a):

Suppose you aim at the top right zone to shoot the ball, what is the probability that you score a goal successfully (i.e. not missing or being saved)?

Question 2(b):

Suggest the probability of scoring a goal successfully when you shoot to the following zone in a penalty kick, ***given that the goalkeeper dives in the same direction as your shot***.

Top Left: _____	Top Center: _____	Top Right: _____
Bottom Left: _____	Bottom Center: _____	Bottom Right: _____

Question 2(c):

If you are a fair (not skillful) player such that the goalkeeper ***can dive in the same direction as your shot*** every time, which zone will you aim at when you take a penalty kick? Explain your answer.

Development:

Ask students to find the probabilities of different events and share their solution

Discuss with the students how they will use the result with their own real-life experience to make the decision in shooting a penalty kick.

Lesson (2): Probabilities about Penalty Kick in Football

Reference: https://www.economist.com/graphic-detail/2018/06/29/penalty-shoot-outs-are-basically-still-crap-shoots

The table below shows the data in the infographic explicitly:

Zone	Goal %	Save %	Miss %
Top Left	75.0%	4.2%	20.8%
Top Centre	75.7%	0%	24.3%
Top Right	74.4%	5.1%	20.5%
Bottom Left	75.6%	18.3%	6.1%
Bottom Centre	67.4%	32.6%	0%
Bottom Right	76.7%	19.5%	3.8%

Reflection:

Is the above data consistent with your results in the pervious tasks?

If possible, you can try to generate an infographic similar to the one above (you may do so by feeding your results into AI) to illustrate your penalty kick statistics.

Development:

Encourage students to reflect on the discrepancy between their results and the real statistics.

Ask students to reflect on their own experience and suggest possible factors that make the discrepancy.

Lesson (2): Probabilities about Penalty Kick in Football

	Reflection in the lesson	Reflection after the lesson
Content-focused reflection	During the in-class discussion, students are encouraged to reflect on • how mathematical knowledge can be used to facilitate a better decision making	Students are encouraged to reflect on • how to extend their experiences in the lesson to continue exploring possible mathematical models to study sport games
Value-focused reflection	During the in-class discussion, students are encouraged to reflect on • the need to study sport through the lens of mathematics and sciences (to avoid becoming a reckless player in sport)	After the class, students are suggested to reflect on • how we can make use of mathematical modelling that could facilitate a better decision making in other real-life scenarios.

Evaluation:

Students will be assessed via

Knowledge	Their works in the worksheet and in-class discussion on how they make use with mathematics to model real-life scenarios
Skill	Their works in the worksheet on how they formulate mathematical model and interpret the result
Attitude	Their engagement in the lesson and their opinion on the usefulness of mathematics in daily life