Wah Yan College, Kowloon

Mathematical Modelling Activities

Shooting on the football pitch

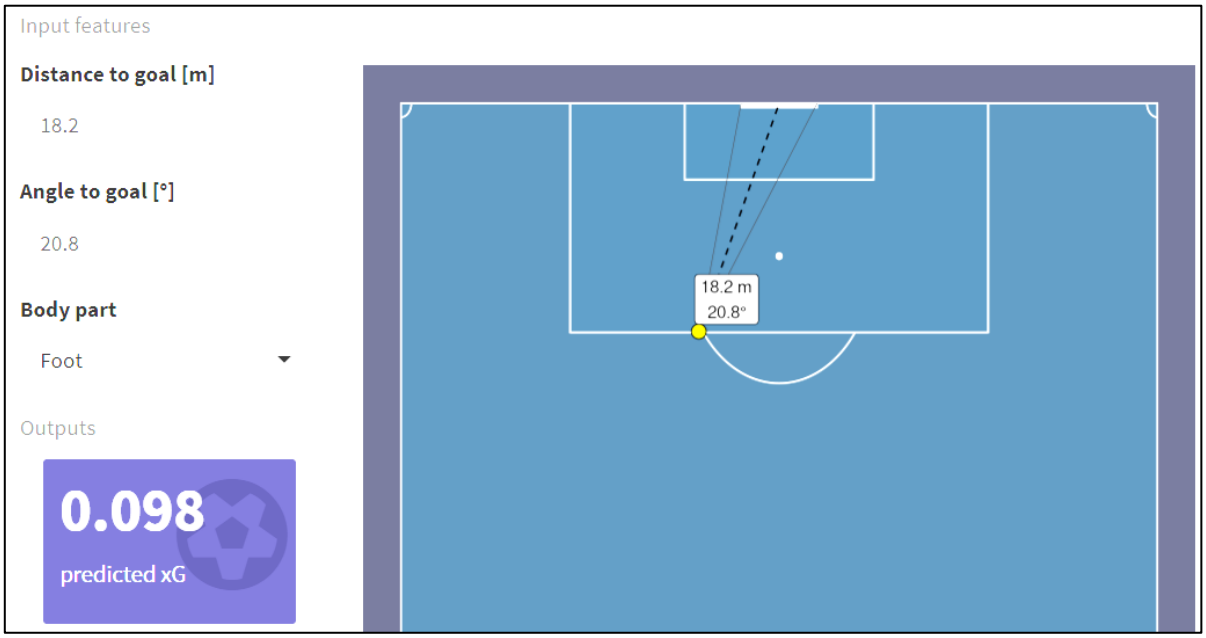
Name: _____ ( ) Class: _____ Date: _____

Task 1: Defining parameters and mathematical model

In this task, we investigate the best strategy for a player to get the greatest possible shooting angle in some scenarios. To make the investigation valid in general scenarios, we should also consider other factors as well (e.g. shooting distance).

Nowadays, some sports scientists use Expected Goals (xG) to measure the chances of a shot successfully entering the goal from a given position on the pitch. This value is based on several factors when the shot is taken.

Here is a demonstration on how xG is calculated: https://www.datofutbol.cl/blog/xg-calculator/

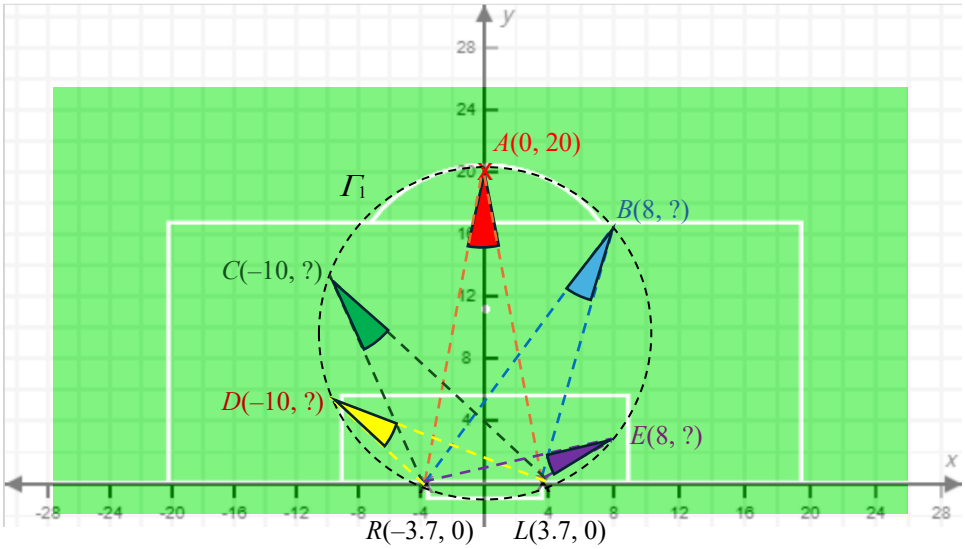


Usually, an xG model uses historical information from thousands of shots with similar characteristics (i.e. make use of big data) to estimate the likelihood of a goal on a scale between 0 and 1. For example, a shot with an xG value of 0.2 is one that we would generally expect to be converted twice in every 10 attempts.

However, now we have no related data to work with. Therefore, we will try a different approach to calculating xG values using coordinate geometry, to see if this approach works or not.

Question 1:

In the figure, Γ_1 is a circle passing through $R(-3.7, 0)$, $L(3.7, 0)$ and $A(0, 20)$. B, C, D and E are points lying on Γ_1 also. The xG values of A, B, C, D and E are provided in the table below.



Shooting point	A	B	C	D	E
xG value provided by data scientists	0.08	0.10	0.13	0.18	0.25
Coordinate	(0, 20)	(8,)	(–10,)	(–10,)	(8,)
θ	21.0°				
d	20				

(a) Complete the above table by finding the corresponding coordinates, values of θ and d of each point.

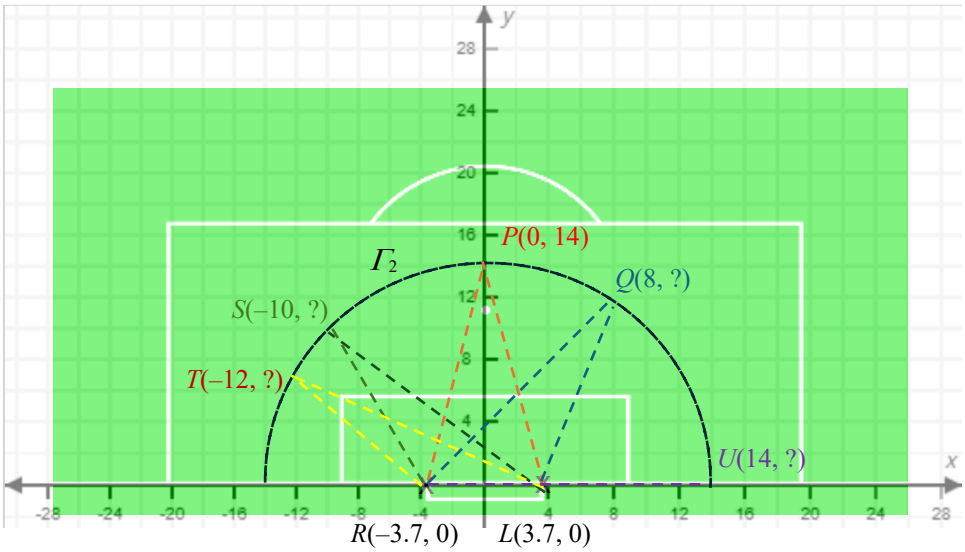
(Hint: You may need to find the equation of Γ_1 first.)

(b) Set up a formula to express the relation between xG and d when $\theta = 21.0°$.

(Hint: You may need to plot the graph of xG against d to study their relation first.)

Question 2:

In the figure, Γ_2 is a circle with origin (0, 0) as its center and radius of 14. P, Q, S, T and U are points lying on Γ_2. The xG values of P, Q, S, T and U are provided in the table below.



Shooting point	P	Q	S	T	U
xG value provided by data scientists	0.181	0.165	0.154	0.138	0.097
Coordinates	(0, 14)	(8,)	(–10,)	(–12,)	(14,)
d	14				
θ	29.6°				

(a) Complete the above table by finding the corresponding coordinates, value of d and θ of each point.
(Hint: You may need to find the equation of Γ_2 first.)

(b) Set up a formula to express the relation between xG and θ when $d = 14$.
(Hint: You may need to plot the graph of xG against θ to study their relation first.)

Question 3:

Based on your findings in Question 1 and 2, Set up a model to express xG in terms of d and θ. (You should determine the relation between xG, d and θ first. Here are some possible relations:

- xG varies jointly as d and θ
- xG is partly constant, partly varies as d and partly varies as θ

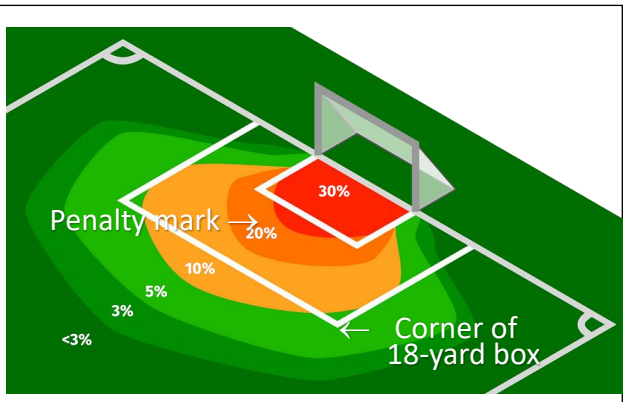
After determining the relation, try to solve the variation constants with the use of data provided in Question 1, 2 or the data generated by https://www.datofutbol.cl/blog/xg-calculator/.)

Note: You can make use of I.T. tools (such as MS Excel) to investigate the problem and search the internet for ***"Multivariate Regression"*** for more information.
You are required to write down the result and no need to show your working at here.

Task 2: Solving mathematical problems and validating solution

Question 4: According to your model, find the xG values if a player shot at

(a) penalty mark (0, 11),
(b) corner of 18-yard box (20.2, 16.5).



With the reference of above figure which shows the xG values on the football pitch (30% means xG of 0.3), can your model give reasonable xG values in (a) and (b)?

Task 3: Drawing conclusions and suggesting further improvement

Question 5: Is your model valid in every scenario? If not, please suggest some ways to further improve your mathematical model.