Suggested Solution

Wah Yan College, Kowloon
Mathematical Modelling Activities
Ramps on our campus

Name: ____________________________ Class: ________ ( ) Date: _____________

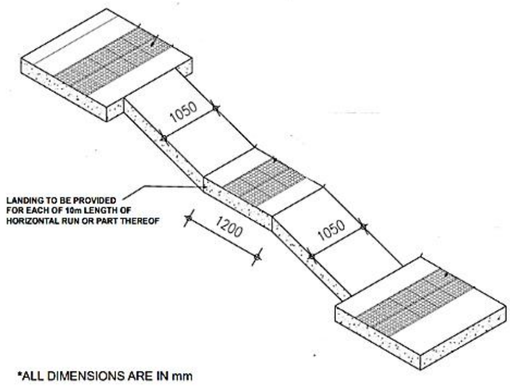
Introduction

Have you tried to use the ramp besides St. Ignatius Chapel? In your opinion, is it a good ramp in design?

First, the ramp besides St. Ignatius Chapel has a gradient less than 1:12 and width of at least 1.05 m. Also, the ramp has provided with horizontal platform for every 10 m of inclined roads, which fulfils the safety regulation in Hong Kong.

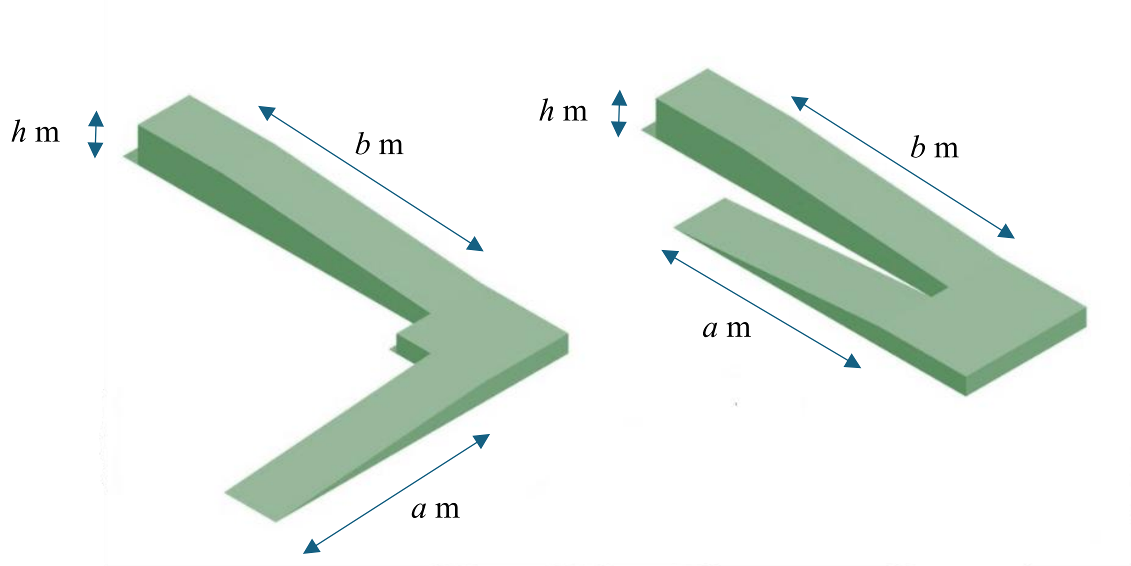
(Source: https://www.bd.gov.hk/doc/en/resources/codes-and-references/code-and-design-manuals/BFA2008_e.pdf)

However, how can we justify whether a ramp is good in design besides the safety factors?



Task 1.1: Defining the question

The figure below shows the two ramps, A and B, with both fulfil the safety regulation:



Ramp A (turning) Ramp B (switchback)

Each ramp consists of two inclined planes with horizontal distance a and b (in metres) respectively, and their total vertical distance is h (in metres). In order to fulfil the safety regulation, each ramp has a horizontal platform between the two inclined planes.

Question 1: Which ramp, A or B, has a better design?
Please state clearly the factor(s) you have considered.

Note that both ramps A and B have

- the same height h m; and
- the same total travel distance $(a + b)$m
  (Assume that the distance travelled on the horizontal platform is relatively small compared to the total distance travelled, so that it can be neglected.)

In order to compare the designs, the difference between ramps A and B should be considered. The most obvious difference between the two ramps is the turning angle on the horizontal platform, which is 90° for ramp A and 180° for ramp B.

A larger the turning angle on the horizontal platform results in

- more time being used to travel the ramp, as it is necessary to slow down in order to pass the sharp turn safely
- more difficult for wheelchair users to travel the sharp turn

Since ramp A has a smaller turing angle, it is suggested that ramp A has a better design.

(OR other reasonable answers)

Task 1.2: Defining parameters and mathematical model

Now let us consider the time required for users to travel up the ramp with wheelchairs.
The travelling speed with different gradients of the ramp are suggested below:

Gradient of the ramp	1 : 20	1 : 15	1 : 12
Speed (ms^{-1}) for the user to travel the ramp upwards	2	1	0.5

Question 2: In addition to the above data, do you require further information for you to calculate the time required for users to travel up the ramp with wheelchairs?
If yes, please briefly describe them below.

The time required for the wheelchair users to travel the turn on the horzontal platform, e.g. 15s for a 90° turn and 30s for a 180° turn, or it can be expressed as $t = 15 \times \frac{\theta}{90°}$, where t is the time required (in s) and θ is the turning angle.

Now you need to use the above information to estimate the time required (in second) for users with wheelchairs to travel the ramp upwards

Question 3: With reference to the above information, set up formulas to estimate the time required (in second) for users with wheelchairs to travel up Ramp A (turning) and Ramp B (switchback).

(Note: It will be great if you can set up a formula which can be used to estimate the travelling time in both ramps.)

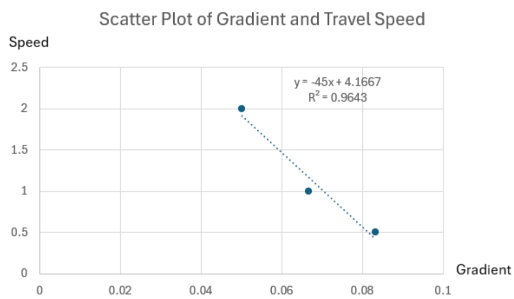
Gradient of the ramp (excluding the horizontal platform) = $\frac{h}{a+b}$

Since the gradient of the ramp may not be exactly equal to $\frac{1}{20}$, $\frac{1}{15}$ or $\frac{1}{12}$, a formula is needed to be set up to calculate the travel speed for the other gradients.

One of the possible formulas is

$y = -45x + 4.1667$

where x is the gradient
and y is the travel speed (in ms^{-1})



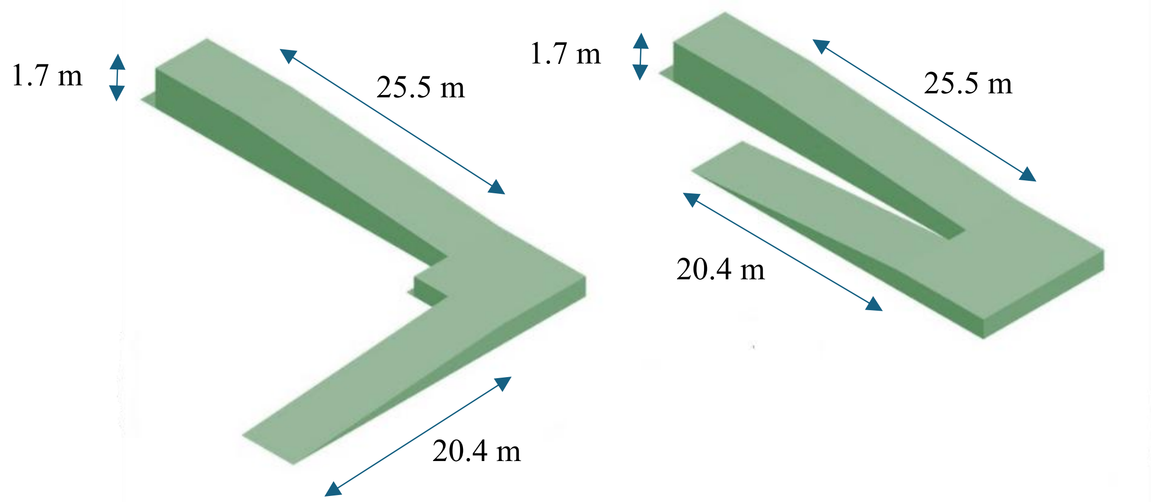
Therefore, the time required

$$= \frac{\text{total distance of the ramp}}{\text{speed of traveling the ramp}} + \text{time required for traveling horizontal platform}$$

$$= \frac{a+b}{-4.5\left(\frac{h}{a+b}\right) + 4.1667} + 15 \times \frac{\theta}{90°} \text{ second}$$

Task 2.1: Solving mathematical problems

Question 4:
Consider the ramp A and B below. Using your formulas, estimate the time required (in second) for users to travel the ramp upwards.



	Ramp A (turning)	Ramp B (switchback)
Time required to travel upwards	$\dfrac{20.4+25.5}{-45\left(\dfrac{1.7}{20.4+25.5}\right)+4.1667}+15\times\dfrac{90^\circ}{90^\circ}$ ≈ 33.4 second	$\dfrac{20.4+25.5}{-45\left(\dfrac{1.7}{20.4+25.5}\right)+4.1667}+15\times\dfrac{180^\circ}{90^\circ}$ ≈ 48.4 second

Task 2.2: Drawing appropriate conclusions

Question 5: Which ramp, A or B, has a better design? Explain your answer.
(Note: Would you consider other factors besides the travelling time?)

Note that the travelling time of ramp B is much higher than that of ramp A, so ramp A has a better design.

(OR other reasonable answers)

Extra Task:
Try to justify whether the ramp besides St. Ignatius Chapel is good in design.

Remark:

While the dangers of steep ramps are well known, ramps with sharp turns can also be dangerous for wheelchair users. Many power wheelchairs today are extremely large and bulky, especially for users with bags and other items hanging from the back. Attempting to navigate a sharp turn in such a chair can result in the user becoming stuck or being forced to go back down the ramp backwards. Therefore, it would be more difficult and time consuming for wheelchair users to make a 180° turn.

Therefore, most of the ramps will avoid a sharp turn (e.g. 180°) in their design. The ramp besides St. Ignatius Chapel is one of the examples. It consists of three 90° turns but no 180° turns.

Please note that sharp turns are more hazardous for wheelchair users travelling downhill. Users must slow down when approaching sharp turns in order to travel safely; otherwise, accidents may occur. This scenario can easily be understood by reference to a similar one, such as a racing car taking a hairpin corner in the Macau Grand Prix.

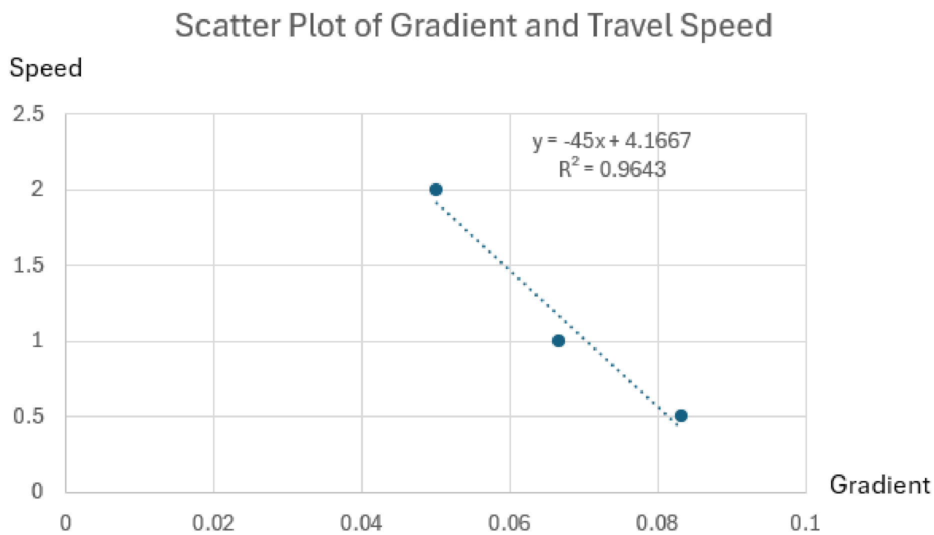
Source - https://www.macau.grandprix.gov.mo/en/news/event/1647

In Question 2, students should consider the difference between a 90° turn and a 180° turn. Since the question considers wheelchair users travelling up the ramp, we can ignore the need for them to slow down in order to pass through the turn safely. Furthermore, students could suggest a linear relationship between the the time required for the wheelchair users to travel the turn on the horzontal platform (e.g. $t = 15 \times \frac{\theta}{90^\circ}$). However, it is still acceptable if students only suggest the time required for a 90° turn (e.g. 15 seconds) and a 180° turn (e.g. 30 seconds), since turns at other angles are not commonly seen.

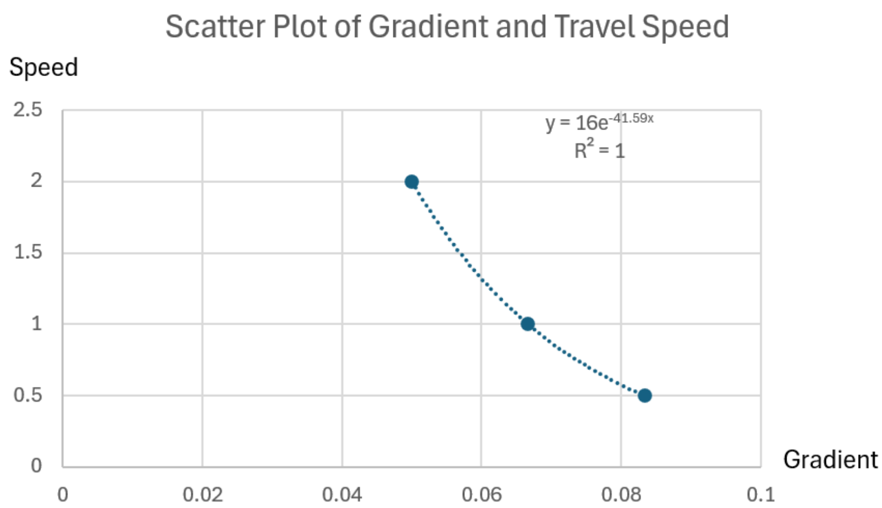
In Question 3, students are required to think about how to find the travelling speed for the user to travel the ramp upwards with gradients other than 1 : 20, 1 :15 and 1 : 12, in order to estimate the speed for the user to travel the ramp upwards with gradients $\frac{h}{a+b}$.

Gradient of the ramp	1 : 20	1 : 19	1 : 18	1 : 17	1 : 16	1 : 15	1 : 14	1 : 13	1 : 12
Speed (ms^{-1}) for the user to travel the ramp upwards	2	?	?	?	?	1	?	?	0.5

As a modelling task, students are supposed to suggest a formula (i.e. mathematical model) to express the relationship between the gradient and travelling speed. They can formulate these models directly using IT tools such as MS Excel. For following shows a mathematical model $y = -45x + 4.1667$, where x is the gradient and y is the travel speed (in ms^{-1}).



For able students, teachers can introduce non-linear model (such as exponential function) to them. It may show a better fit to the data.



Finally, students are required to combine the time required for travelling the ramp (excluding the horizontal platform) and making the trun on the horzontal platform in order to build a mathematical model to estimate the total time required. Teachers may invite students to present their models to the class and justify whether their model is reasonable by referring to the results of Question 4 (the models adopted by each student may be different, at least they should get the same result that the time required for travelling ramp A should be smaller than that of ramp B).

After the task, teachers could encourage student to keep further exploration on their living community to see whether sufficient ramps are built for the people with needs and also examine whether the ramps built in the campus and their living community are good in design with the use of mathematics. This is one of the major objectives of mathematical modelling education – using mathematics to understand and shape our world.